Analysis and design of large scanning arrays made possible by the use of accurate equivalent circuit models

Roberto Vincenti Gatti

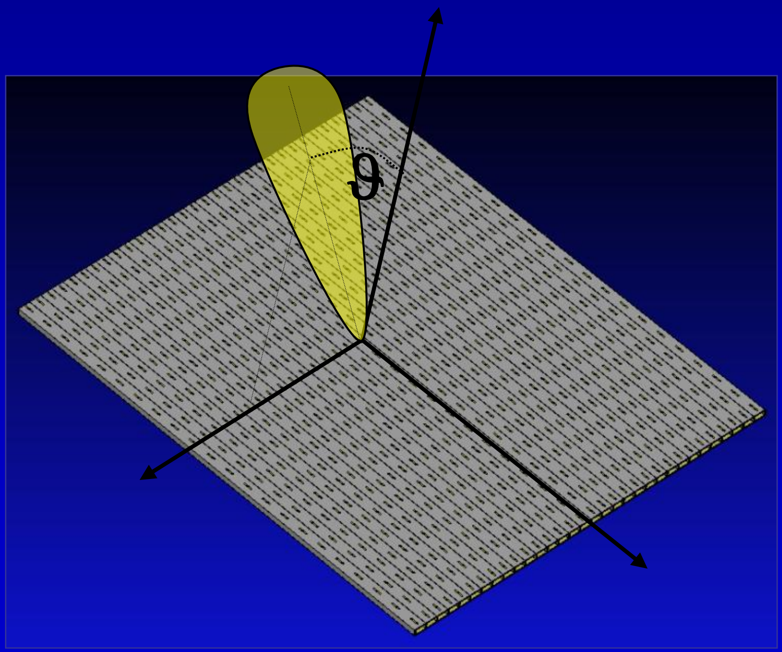
Outline

- Introduction to large scanning arrays
- Use of equivalent circuit models:
  - the radiating element
  - the sub-array (slotted WG)
  - the input transition
- Design method of the array
- Analysis method of the array
- Results
- Conclusions

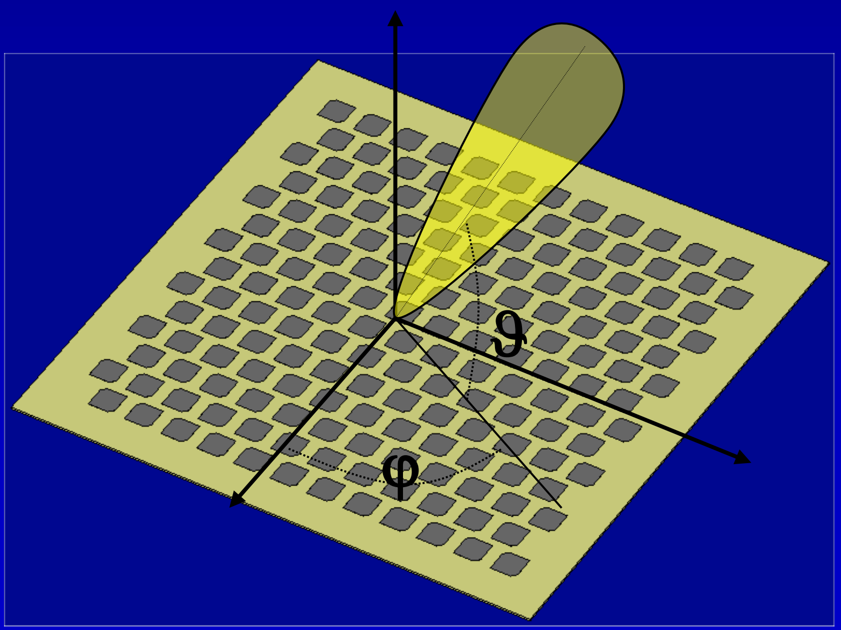
Introduction

Large scanning arrays applications:

- radar systems
- communication systems
- mobile satellite systems



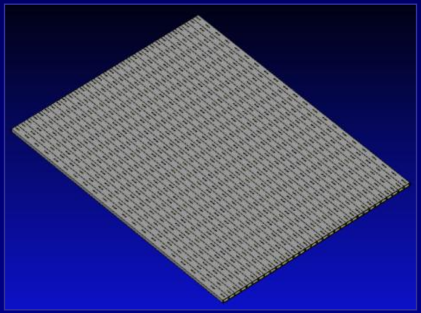
Array of slots / 1-D scanning

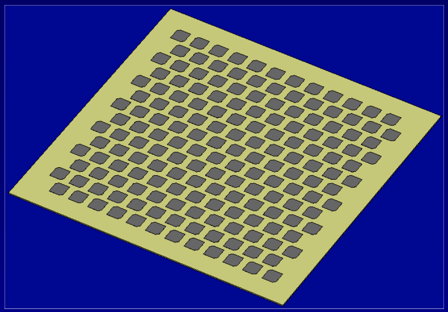


Array of patches / 2-D scanning

Introduction

Analysis and design of large arrays made of hundreds or thousands of radiating elements





Computation unaffordable by any full-wave simulator

Simplified approach based on:

- Equivalent circuit of the radiating element based on accurate full-wave analysis
- Mutual couplings lumped in the model using the "active impedance" concept

Fast and accurate analysis and design of large arrays

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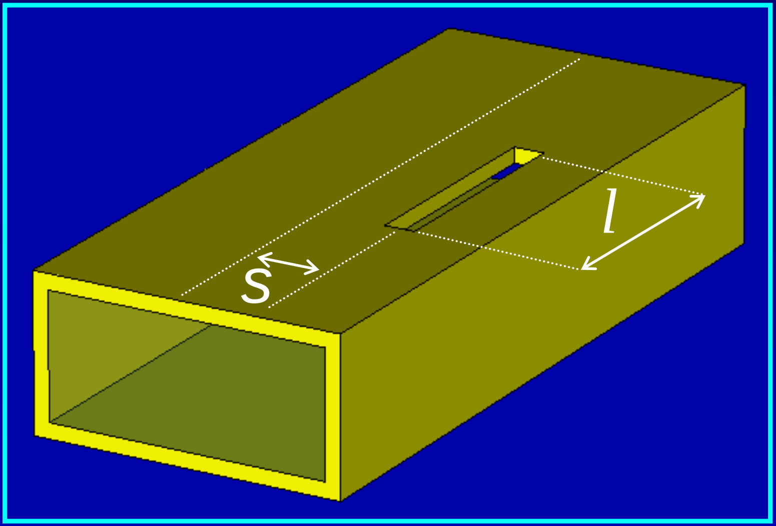
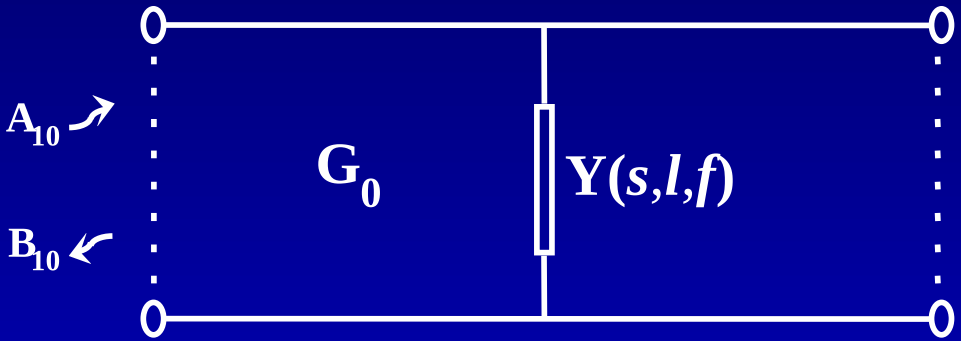
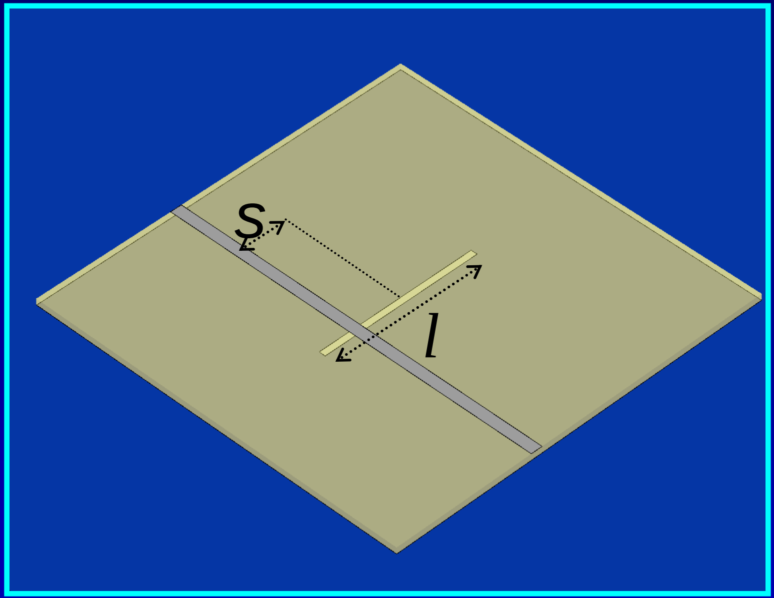
Equivalent circuit: slotted WG

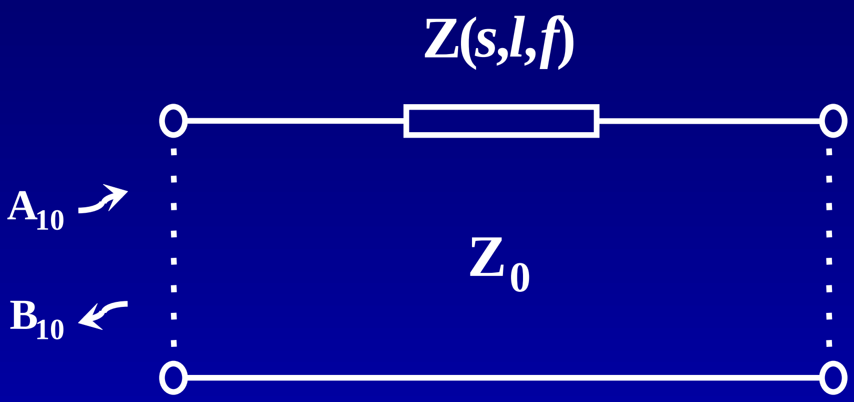




$$S_{11} = S_{22} = -\frac{Y/G_0}{2 + Y/G_0} \quad \Rightarrow \quad \frac{Y(s,l,f)}{G_0} = -\frac{2S_{11}}{1 + S_{11}}$$

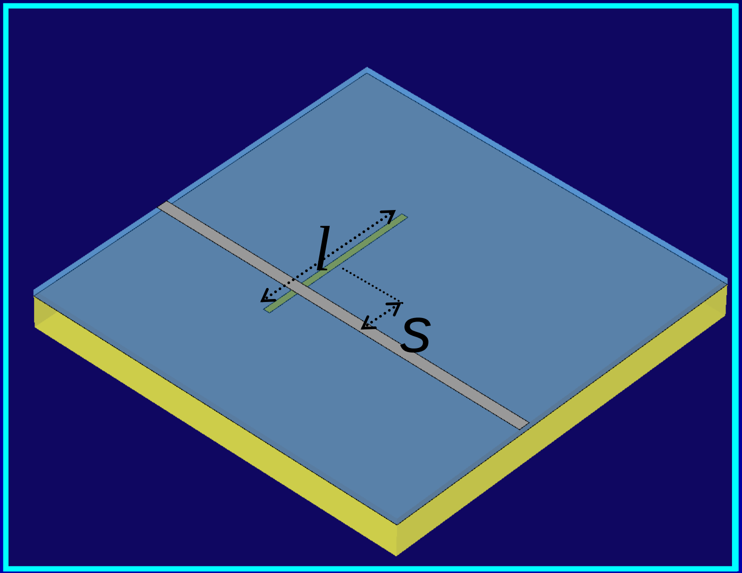
Equivalent circuit: MS-fed slot

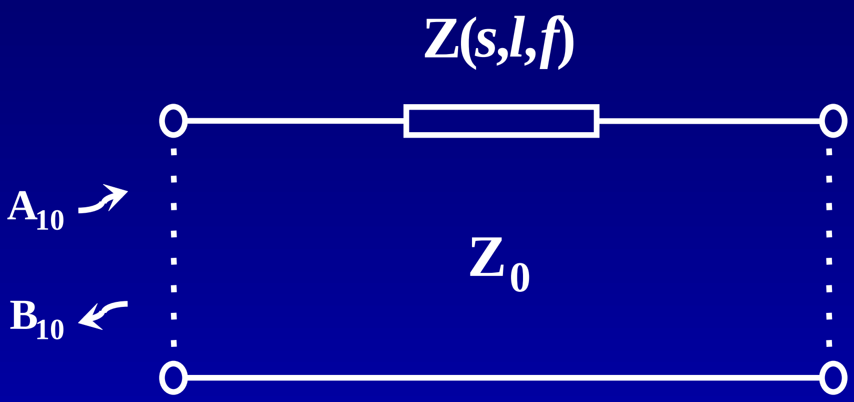




$$S_{11} = S_{22} = \frac{Z/Z_0}{2 + Z/Z_0} \quad \Rightarrow \quad \frac{Z(s,l,f)}{Z_0} = \frac{2S_{11}}{1 - S_{11}}$$

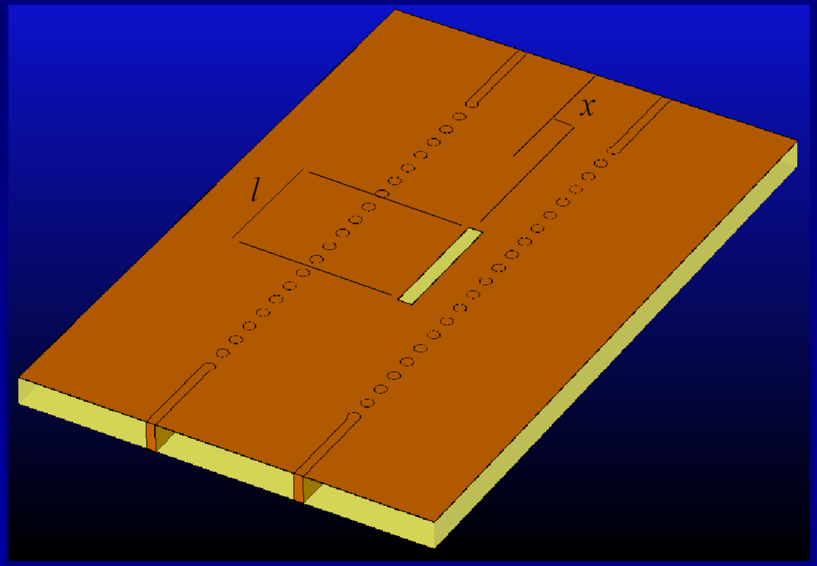
Equivalent circuit: MS-fed patch

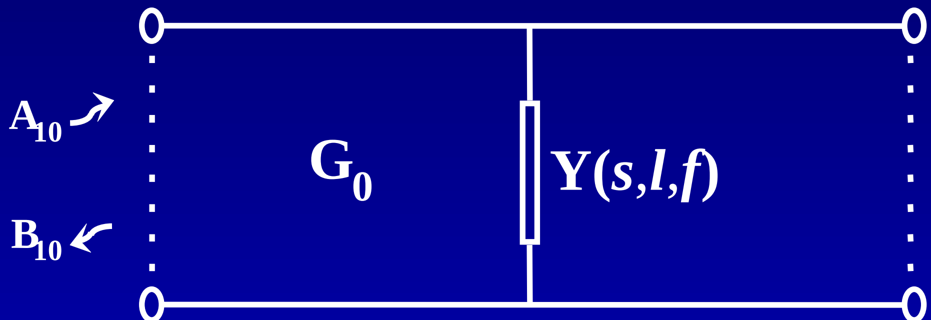




$$S_{11} = S_{22} = \frac{Z / Z_0}{2 + Z / Z_0} \quad \Rightarrow \quad \frac{Z(s,l,f)}{Z_0} = \frac{2S_{11}}{1 - S_{11}}$$

Equivalent circuit: SIW slotted WG





$$S_{11} = S_{22} = -\frac{Y/G_0}{2+Y/G_0} \Rightarrow \frac{Y(s,l,f)}{G_0} = -\frac{2S_{11}}{1+S_{11}}$$

Computation of the slot admittance

Factorization of the equivalent admittance:

Slot admittance is expressed as a combination of functions of ***one variable*** $\rightarrow$ FAST - LOW MEMORY

Admittance normalized to resonant conductance

$$\frac{Y(s,y)}{G_0} = \frac{G_r}{G_0} \cdot \frac{G + jB}{G_r} = g(s) \cdot \left[h_1(y) + jh_2(y)\right]$$

Normalized resonant conductance

$$y = \frac{l}{l_r(s,f)}$$

The normalized resonant length only depends on the offset *s*

$$\frac{l_r}{\lambda_0} = \frac{v(s)}{2\pi} \quad \Longrightarrow \quad l_r(s,f) = \frac{c_0}{2\pi f} v(s)$$

Computation of the slot admittance (2)

1. Compute S_{11} by FDTD (full-wave) simulation for:
   - a set of offsets $\{s_i\}$, $0 < s_i < a/2$
   - length $\bar{l} \approx l_r$

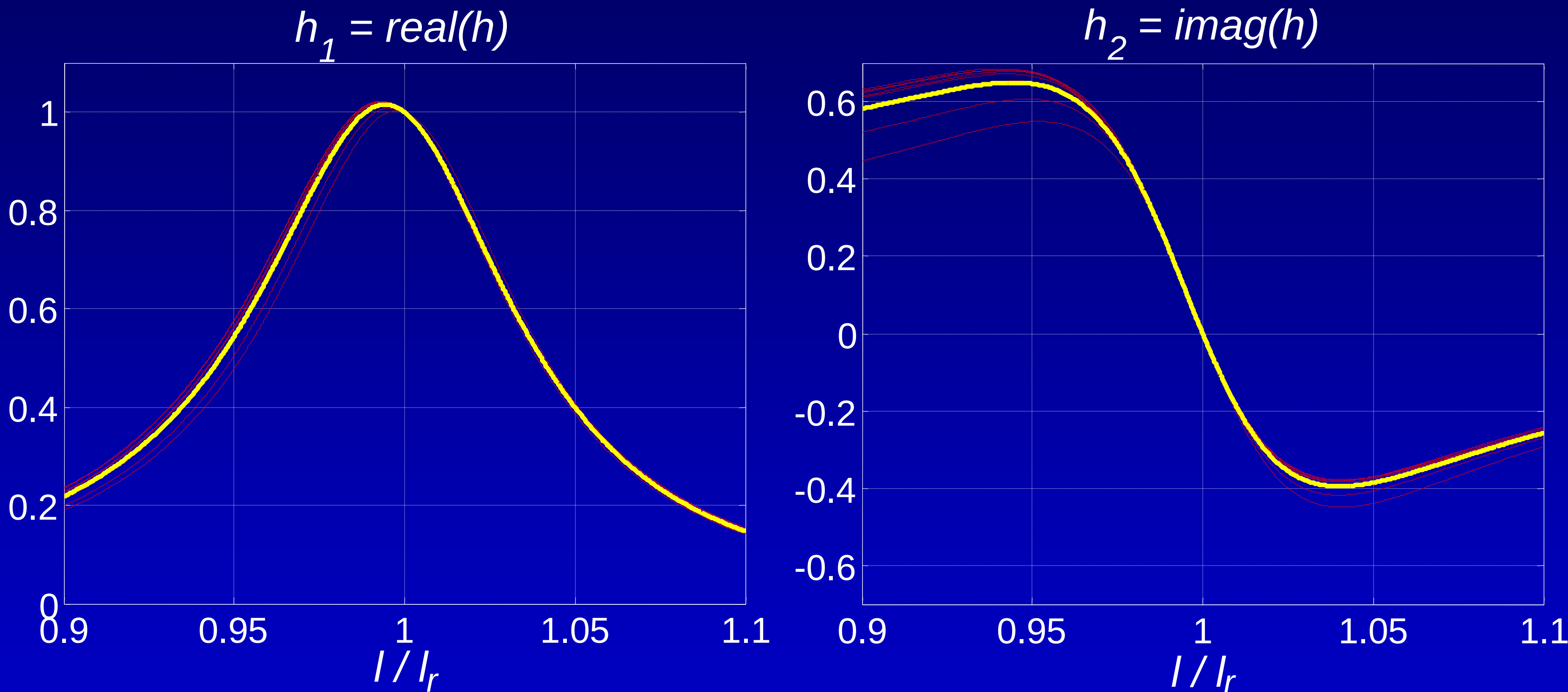
2. At resonance ($f = f_{r,i}$) compute $g(s_i)$ and $v(s_i)$:

$$g(s_i) = \frac{G_r}{G_0} = -\frac{2S_{11,i}(f_{r,i})}{1+S_{11,i}(f_{r,i})} \qquad v(s_i) = \frac{2\pi \bar{l} f_{r,i}}{c_0}$$

3. Off resonance compute $h_{1,i}(y)$ and $h_{2,i}(y)$:

$$h_i(y) = \frac{Y/G_0}{g(s_i)} = \frac{-2S_{11,i}(f_y)}{g(s_i)\left[1+S_{11,i}(f_y)\right]} \; , \quad f_y = y f_{r,i}$$

Air-filled WR90 f_0 = 9.375 GHz

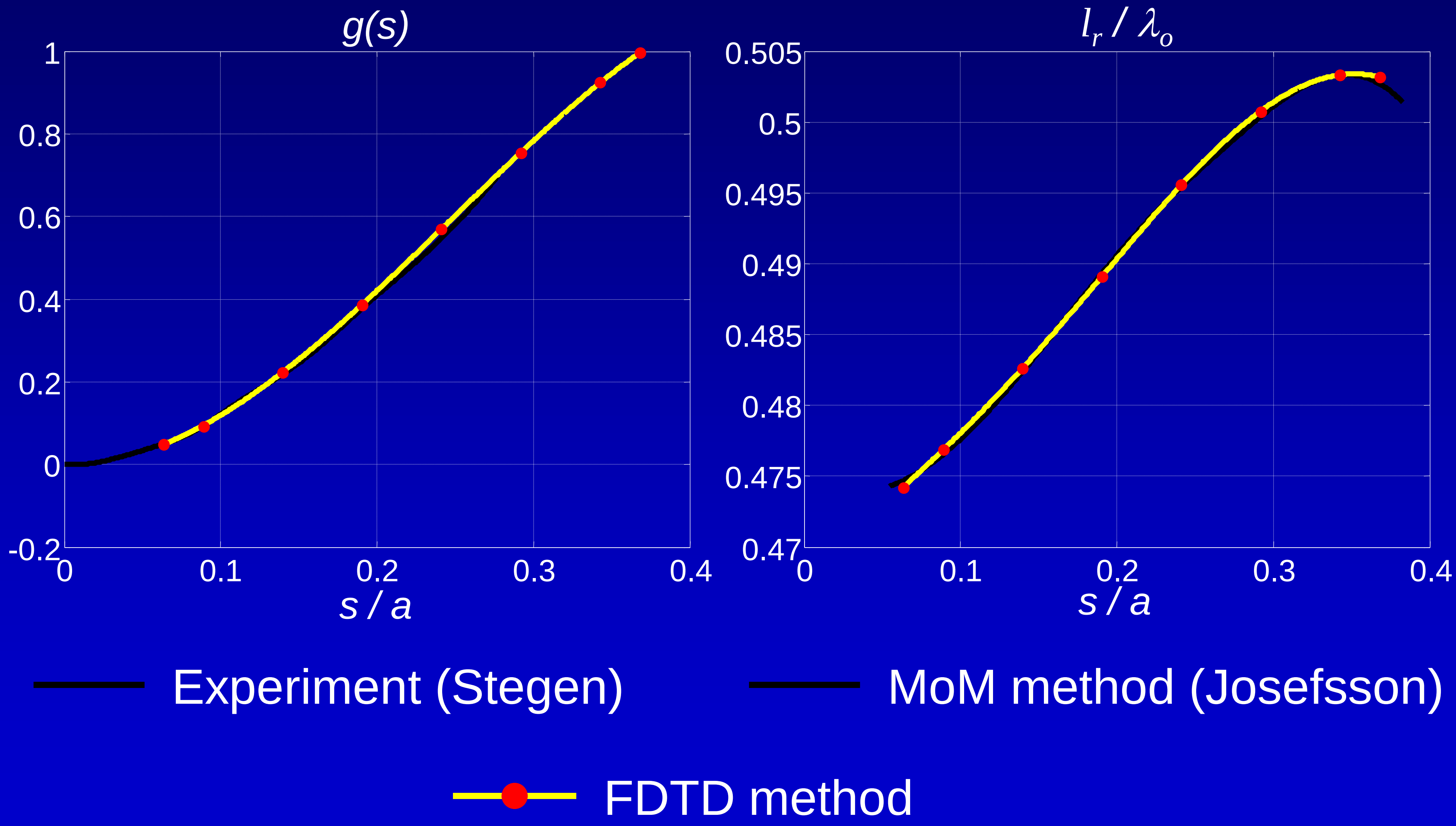


FDTD simulations (offset s_i)

Average of all simulations

Air-filled WR90

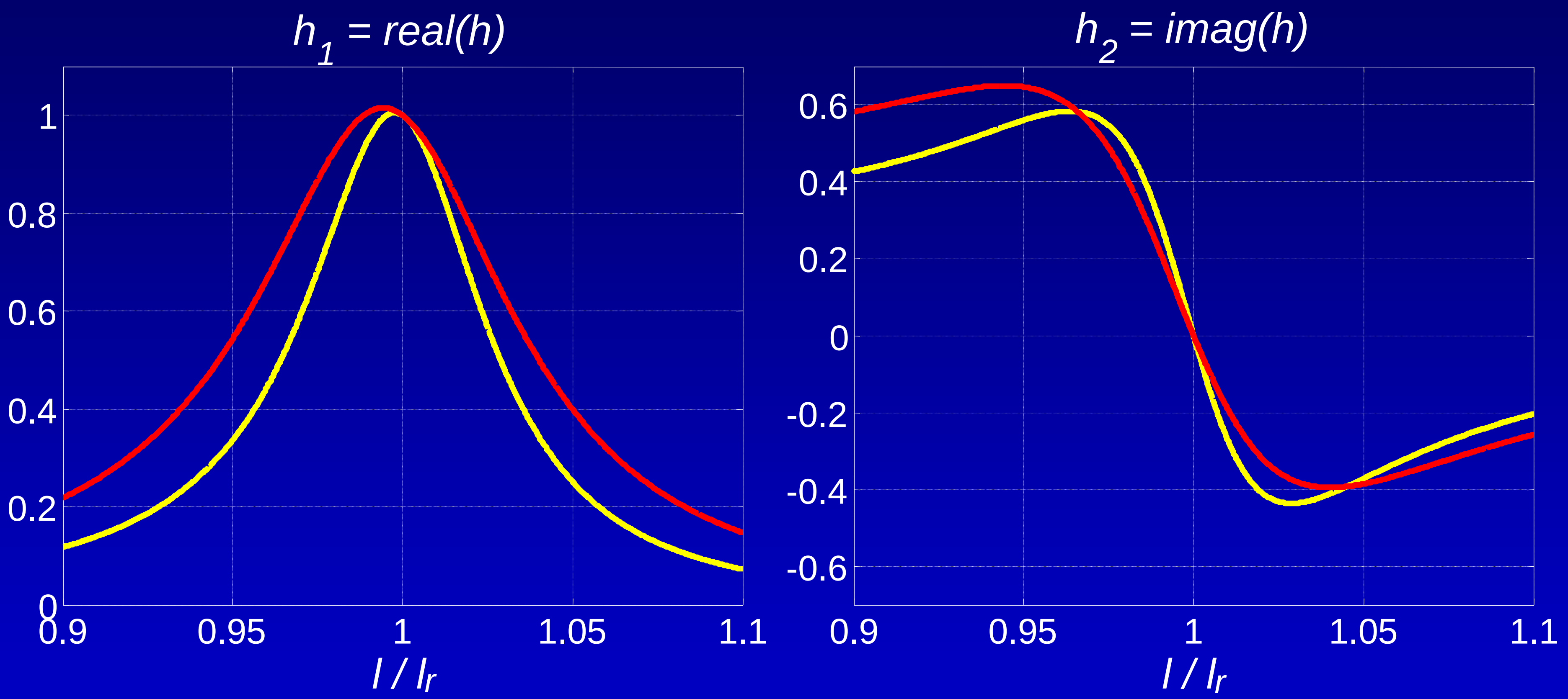
f_0 = 9.375 GHz



Dielectric-filled WR90

f_0 = 6.7 GHz

ε_r = 2.1



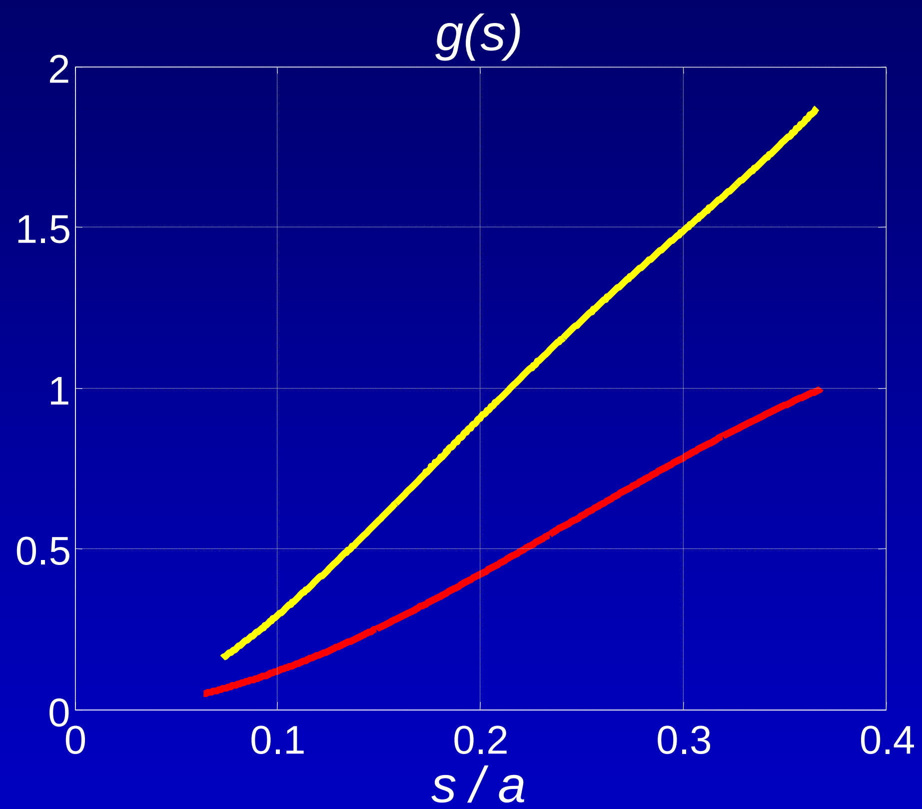
Air-filled WR90

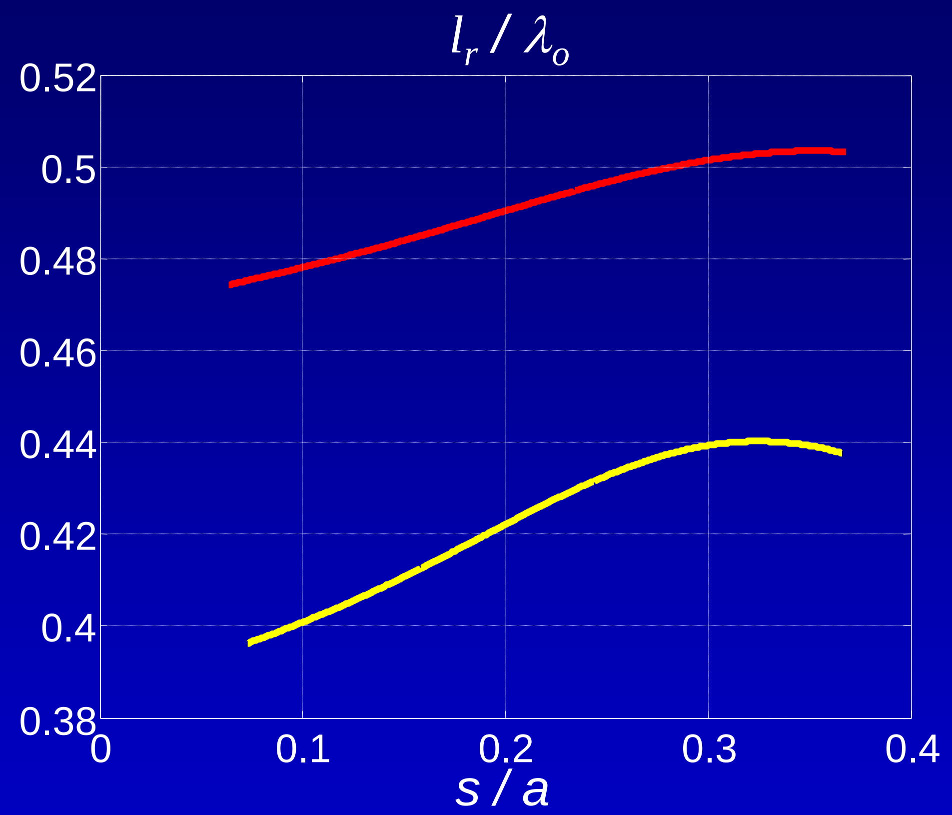
Dielectric-filled WR90 $\rightarrow$ bandwidth reduction

Dielectric-filled WR90

f_0 = 6.7 GHz

ε_r = 2.1





Air-filled WR90

Dielectric-filled WR90

→ resonant conductance increase

→ resonant length reduction

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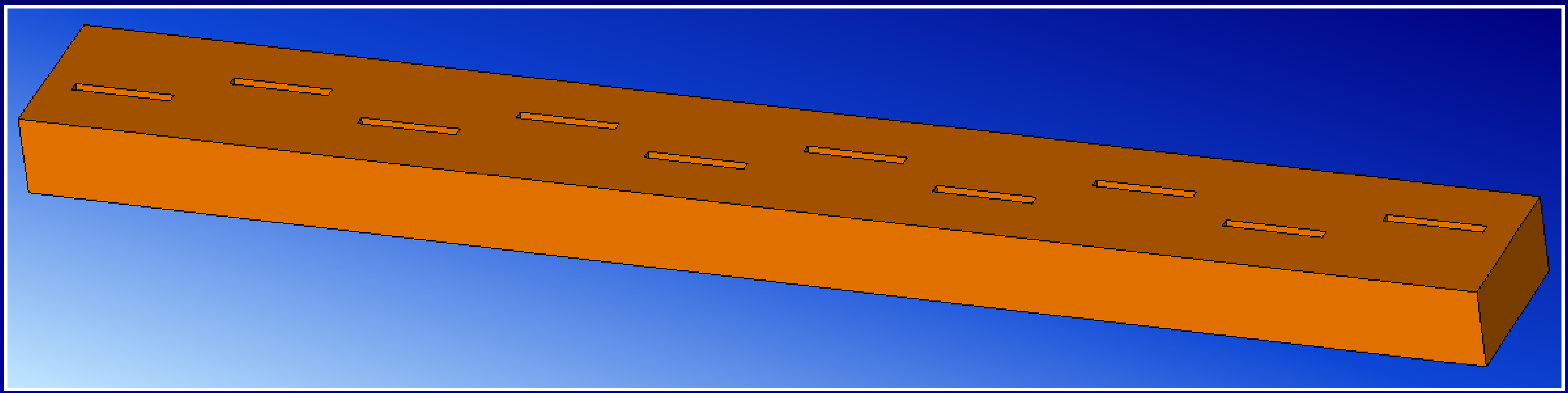
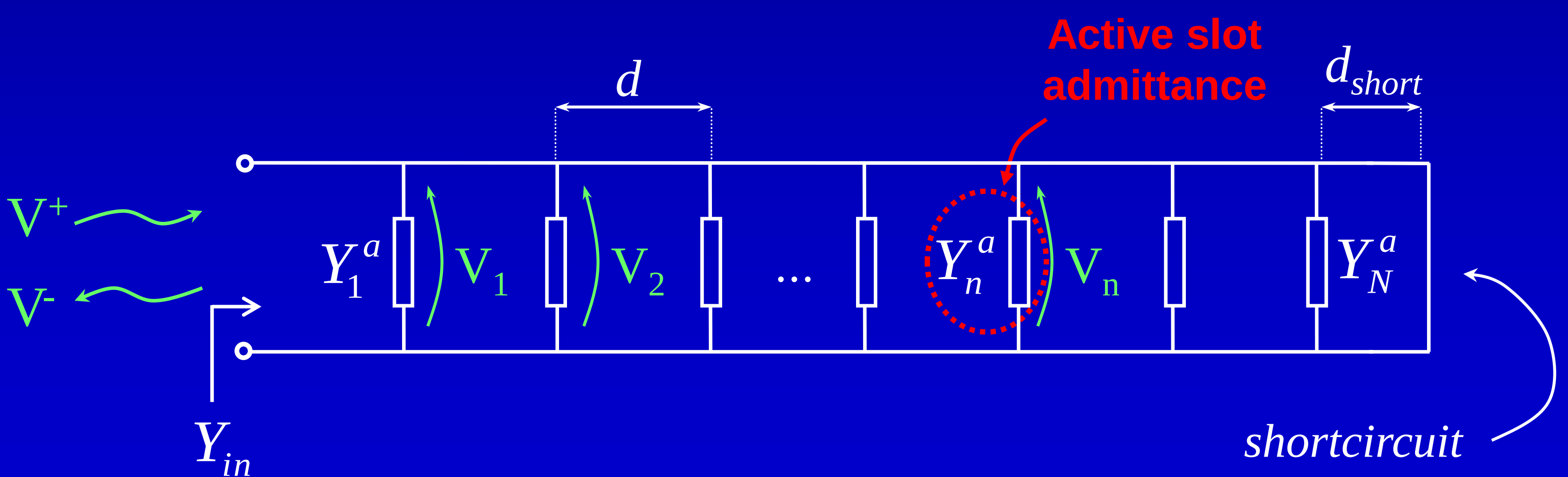
Equivalent circuit of the slotted WG





The "Active Admittance" concept

Admittance matrix of the array

Self admittance

Mutual admittance

$$\bar{I} = \mathbf{Y} \times \bar{V} \quad \Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ \cdots \\ V_N \end{bmatrix}$$

"Active" slot admittance:

$$Y_m^a = \frac{I_m}{V_m} = Y_{mm} + \sum_{\substack{n=1 \\ n \neq m}}^{N} Y_{mn} \frac{V_n}{V_m} = Y_{SELF} + Y_{MC}$$

Isolated slot

Mutual coupling term

Computation of the mutual coupling

The mutual impedances between dipoles can be computed numerically by:

$$Z_{pq} = j\frac{\eta}{4\pi k_0}\int_{-l_p/2}^{l_p/2}\int_{-l_q/2}^{l_q/2}\frac{I_p(z_1)I_q(z_2)}{I_p(0)I_q(0)}\left(k_0+\frac{\partial^2}{\partial z_1^2}\right)\frac{e^{-jk_0R_{pq}}}{R_{pq}}dz_2dz_1$$

(*Vector Potential - Hallén*)

Using the equivalence with an electric dipole, the formula can be extended to any radiating element with linear current/field distribution.

Example: equivalence slot $\leftrightarrow$ dipole

$$Y_{slot} = \frac{2}{\eta^2}Z_{dipole}$$ (*Booker equation*)

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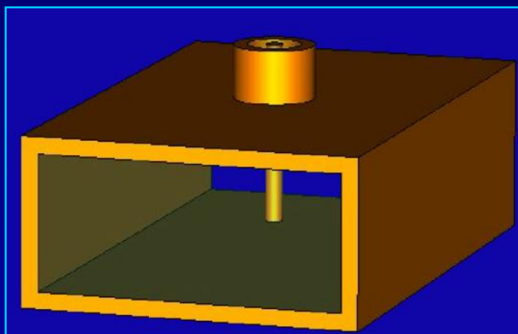
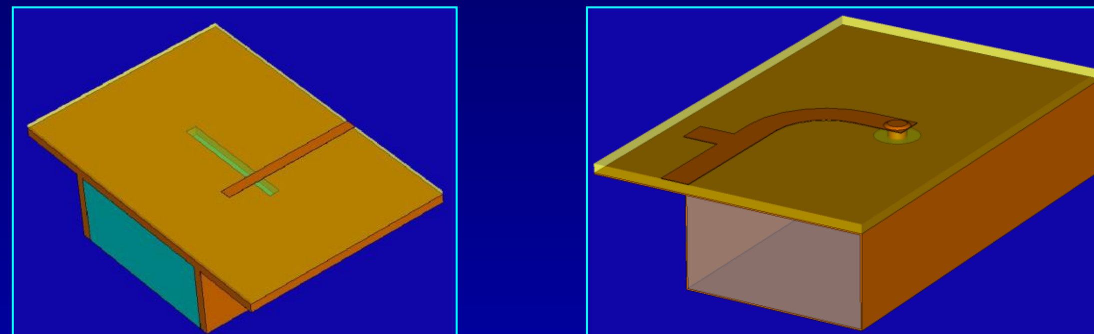
Equivalent circuit of the transition

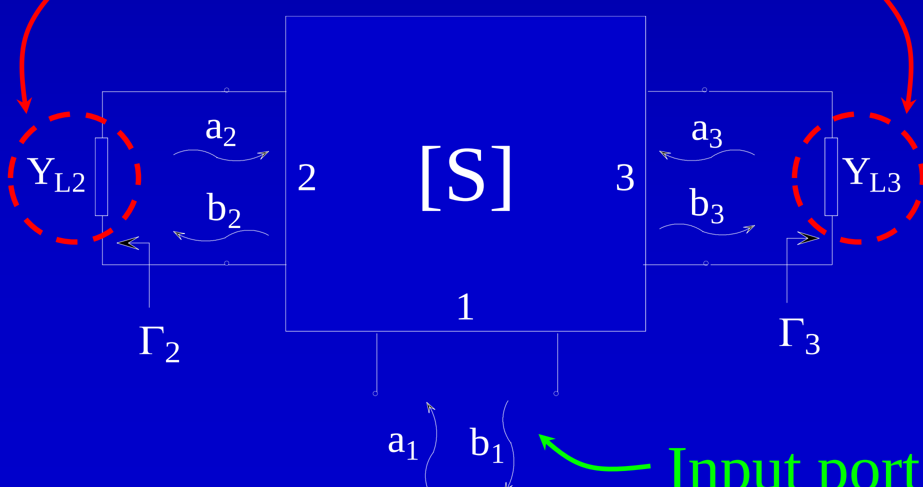
Coaxial-to-Waveguide



Microstrip-to-Waveguide



Slotted waveguide input admittances



Input port

Reflected wave at the input port

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 & -\Gamma_2 S_{12} & -\Gamma_3 S_{13} \\ 0 & 1-\Gamma_2 S_{22} & -\Gamma_3 S_{23} \\ 0 & -\Gamma_2 S_{32} & 1-\Gamma_3 S_{33} \end{bmatrix}^{-1} \times \begin{bmatrix} S_{11} \\ S_{21} \\ S_{31} \end{bmatrix} a_1$$

Waves incident on the waveguides

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Elliott's design method

Mutual coupling effects → *Isolated slots are **not resonant** anymore*
*Degradation of the **radiation pattern***
*Sub-array **mismatching***

⇨ Mutual coupling effects should be included in the design

Iterative procedure:

repeat until convergence is reached

1) Computation of mutual coupling terms
2) Computation of active slot admittances
3) "Detuning" of the slots →
   - Resonance
   - Matching
   - Desired V_{slot}

$$Y_m^a = Y_{SELF} + Y_{MC}$$

R.S. Elliott, *Trans. AP*, 1983

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Analysis Method

Based on Elliott's theory:

1) $$\frac{Y_m^a}{G_0} = K_1 f_m \frac{V_m^S}{V_m}$$

Voltage across Slot admittance

Slot Voltage

2) $$\frac{1}{Y_m^a / G_0} = \frac{1}{Y(s_m, l_m, f_0)/G_0} + \frac{1}{2 f_m^2} MC_m$$

Isolated Slot

Mutual Coupling Term

Eliminating Y_m^a from (1) and (2) ⇨ 3) $$\sum_{n=1}^{N} Q_{mn} V_n^S = K \cdot f_m V_m$$

Slot Voltages (unknown)

Voltages across slot admittances (known)

Analysis Method

Computation of slot voltages $V^S{}_m$ and of input admittances → Iterative Procedure

repeat until convergence is reached

a) Assume active admittances $Y^a{}_m$ as isolated

b) Compute the incident waves into each waveguide

c) Compute the admittance voltages V_m

d) Compute the slot voltages $V^S{}_m$ (from eq. 3)

e) Compute the active slot admittances $Y^a{}_m$

Analysis for different scanning angles → Varying the phases of the incident waves at the input ports

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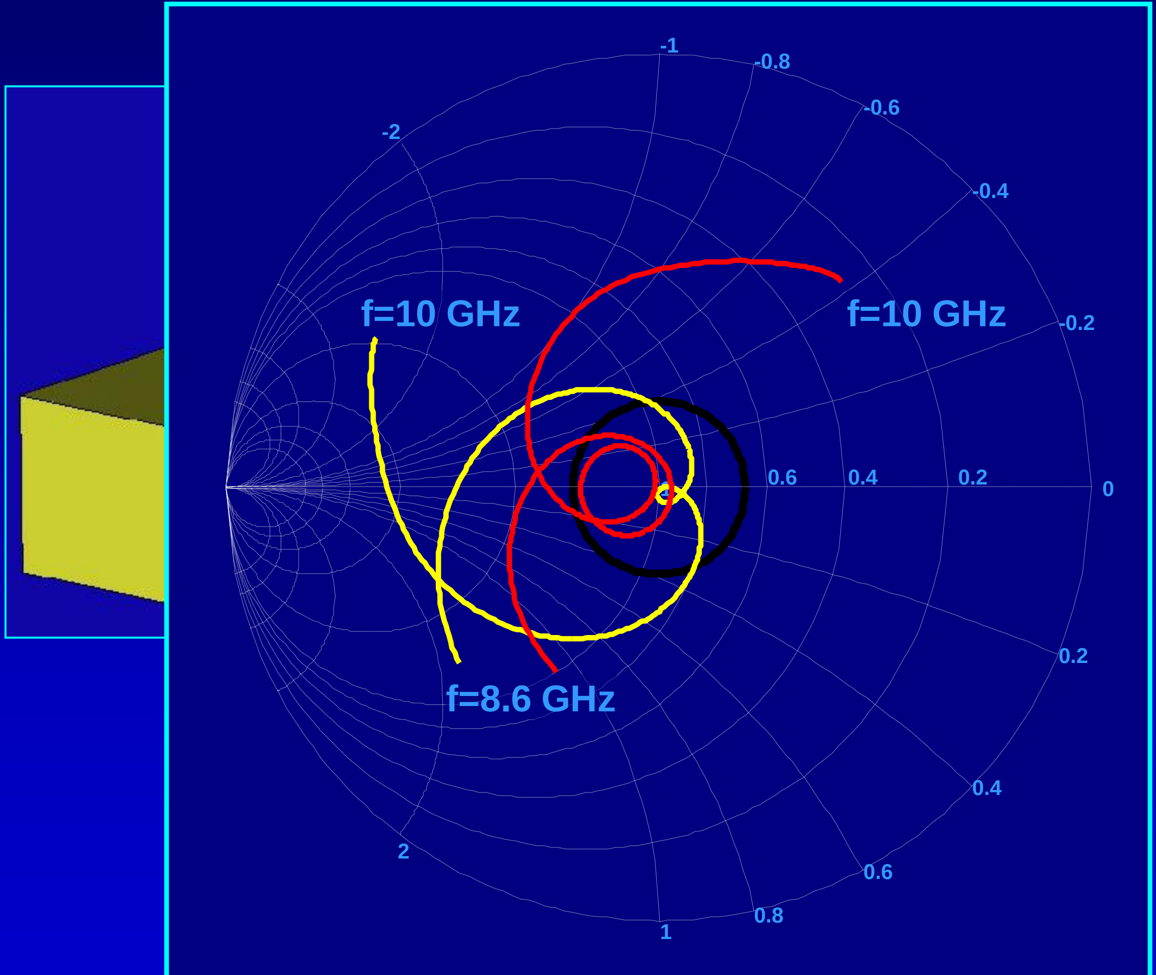
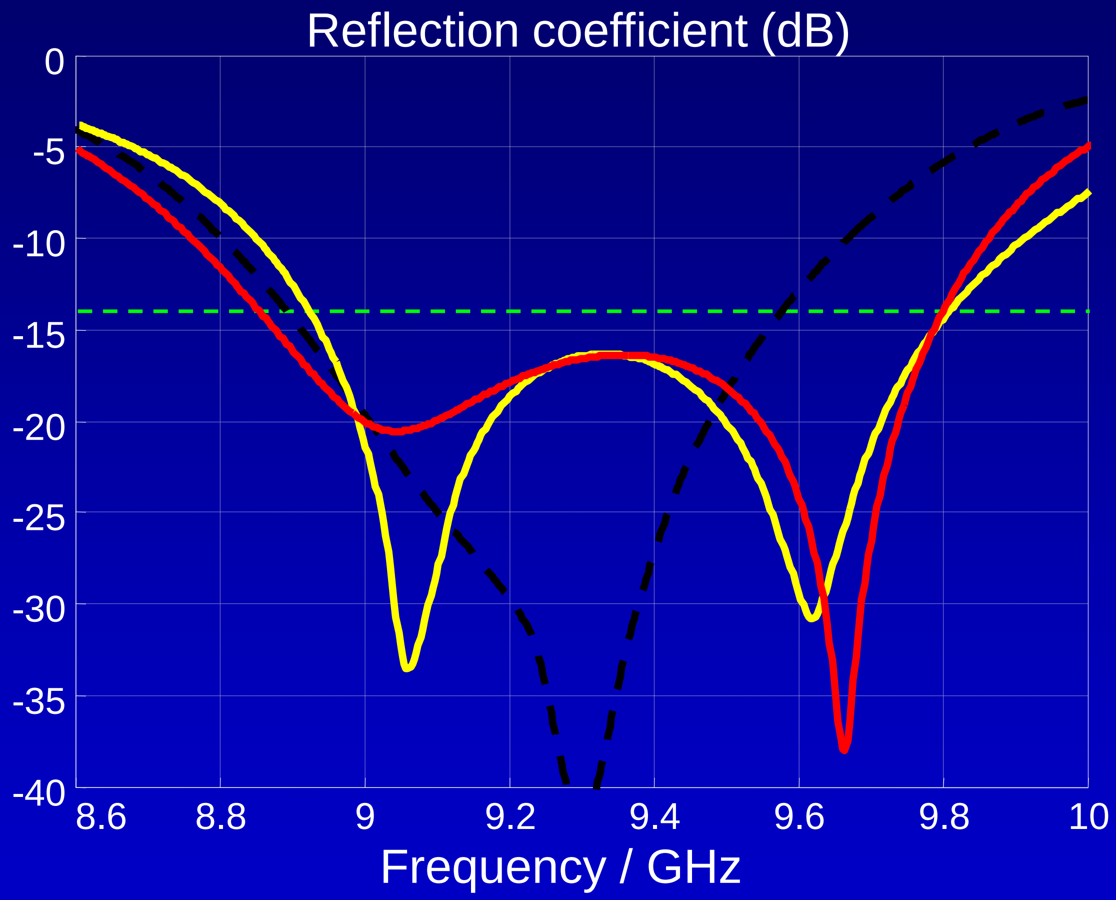
Results: Comparison with a full-wave analysis (CST)



— $Y_{in} = G_0$ — $Y_{in} = 1.3\ G_0$

- WR90 standard
- 4 slot linear array
- Optimised coaxial to waveguide transition
- $f_0 = 9.3$ GHz
- $Y_{in} = 1.3\ G_0$

Results: Reflection Coefficient



Centre frequency:

9.3 GHz

Bandwidth
(with VSWR<1.5):

860 MHz

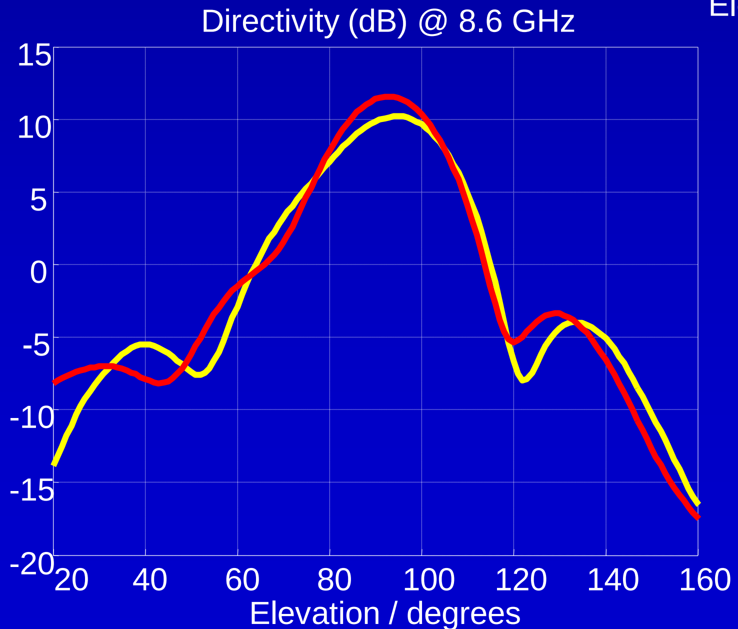
This method

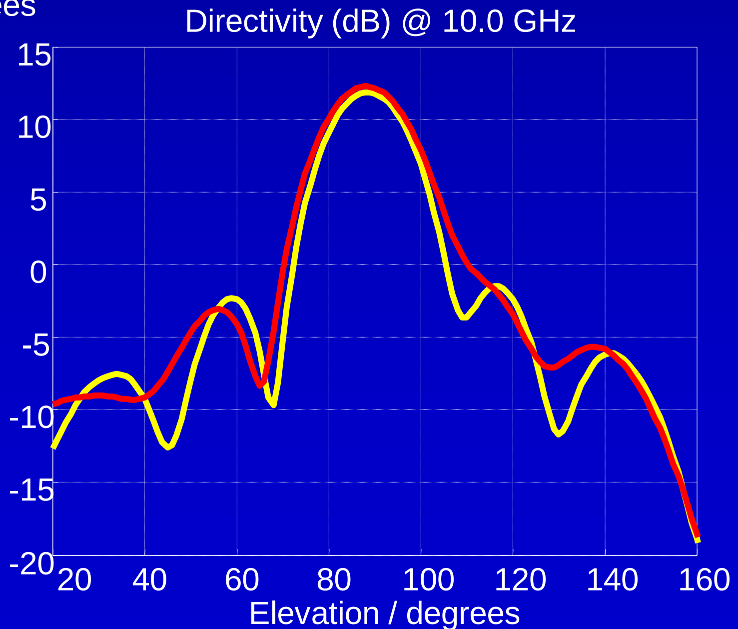
Full-wave analysis

Results: Radiation Pattern

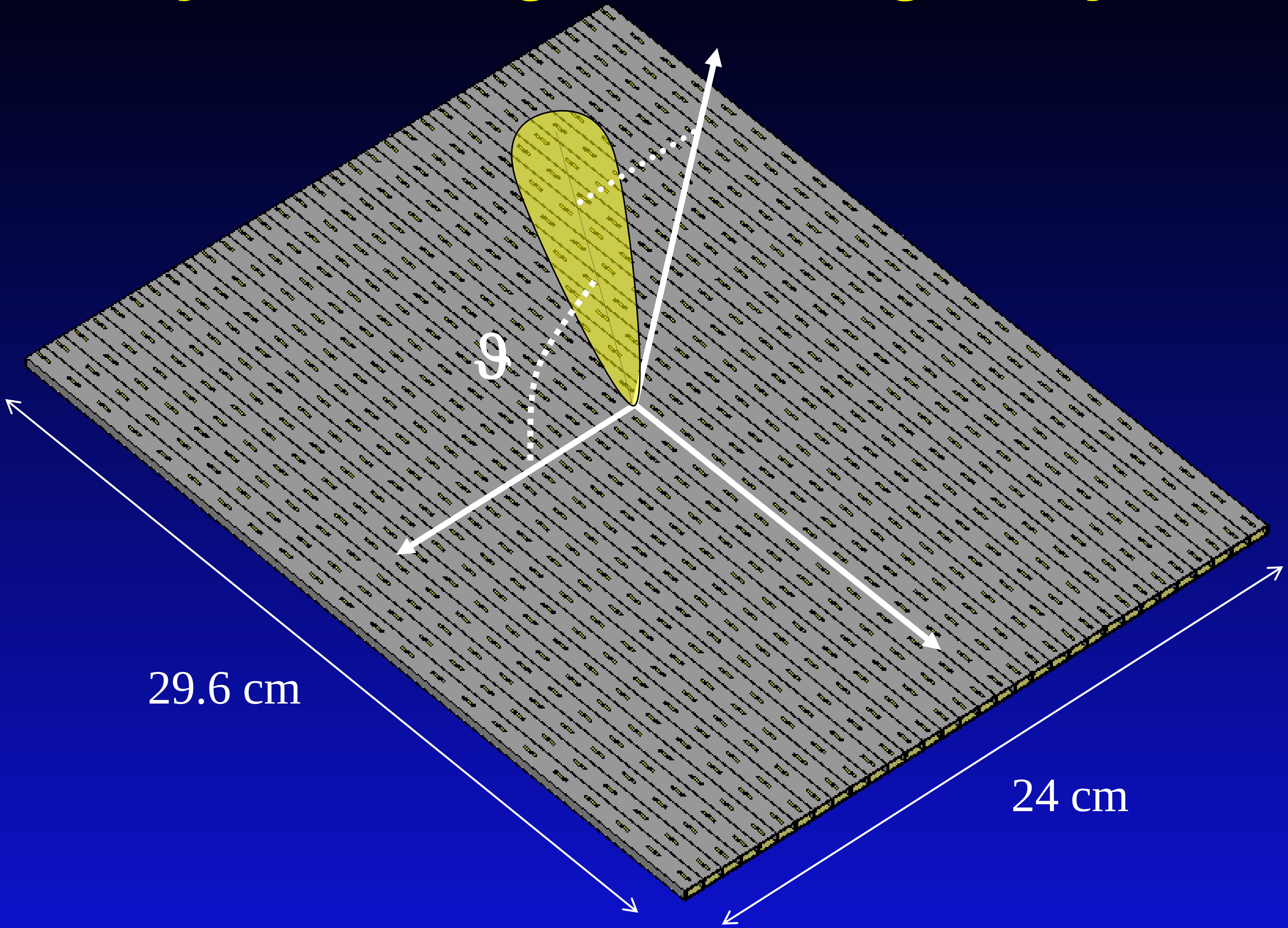
This method

Full-wave analysis

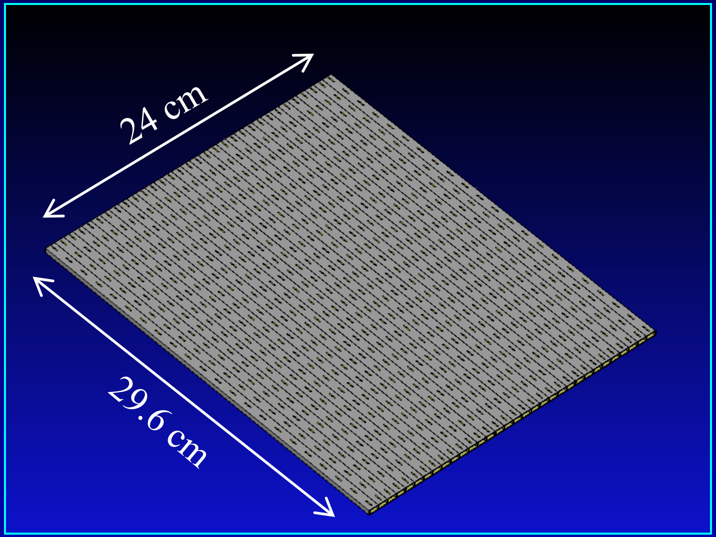




Analysis of large scanning arrays



Analysis of large scanning arrays

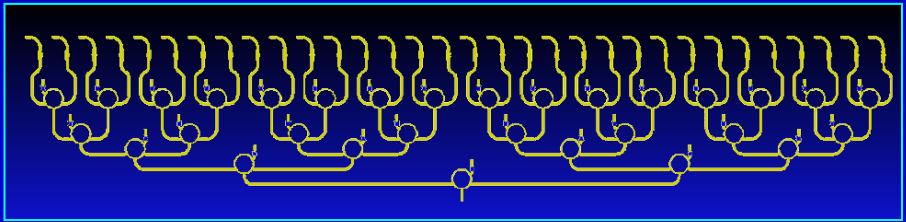


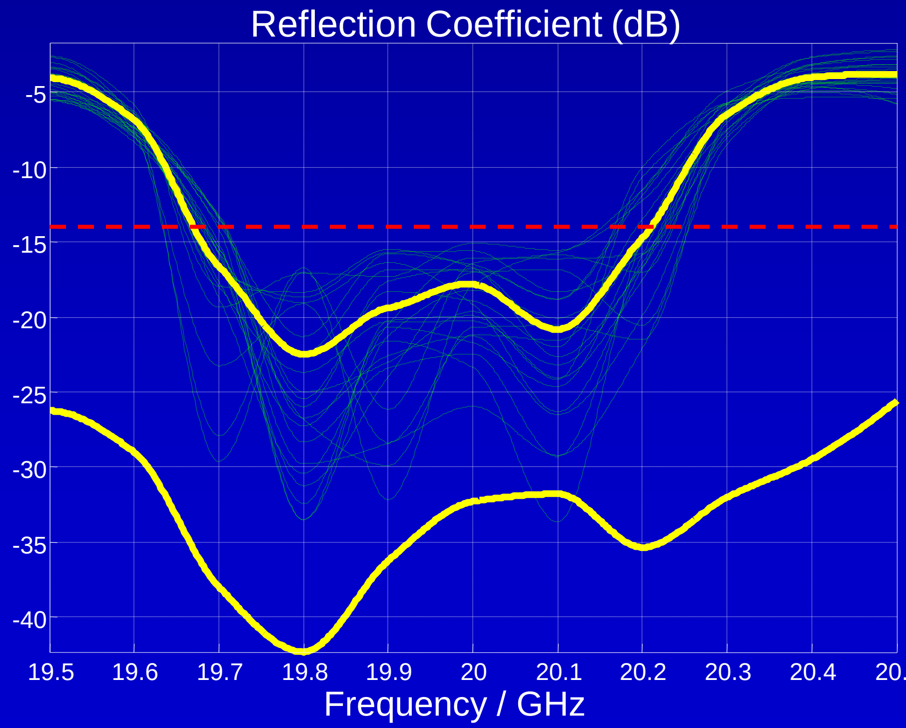
- Array 32 x 22 center-fed
- WG: 7.5 mm x 3.75 mm ($\varepsilon_r = 2.2$)
- $f_0 = 19.95$ GHz ; $\vartheta_0 = 40°$ (elev.)
- Uniform excitation

Bandwidth: **540 MHz** (VSWR<1.5)

Simulation time: **45 min**

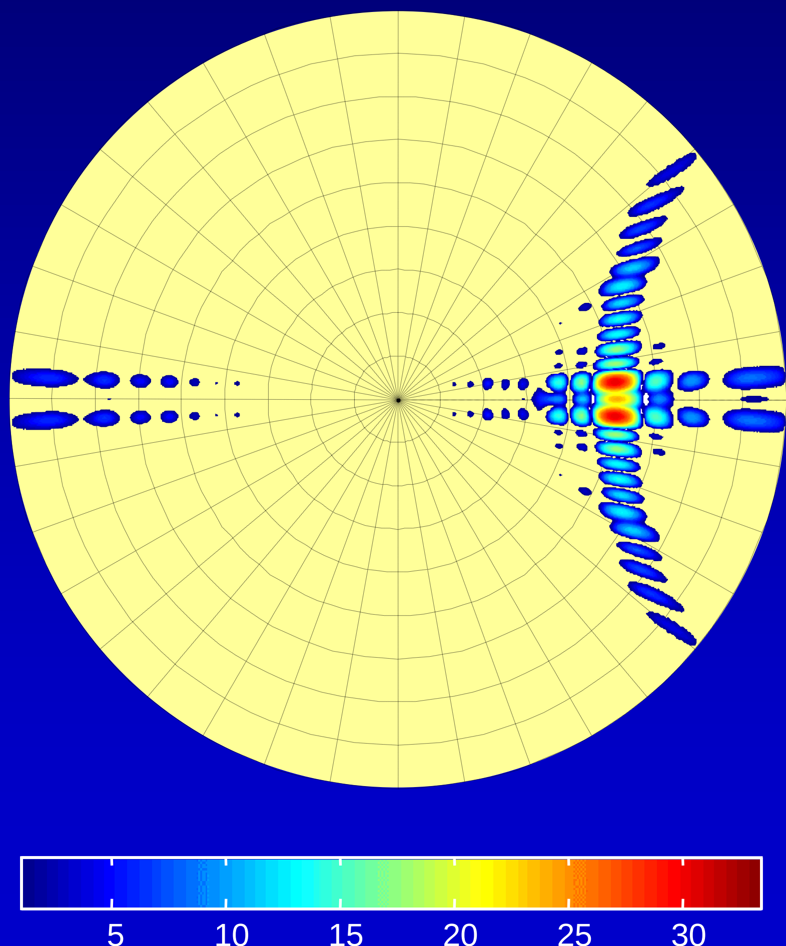
(Analysis performed on a PC with Intel® Pentium® 4 @ 2.5 GHz)





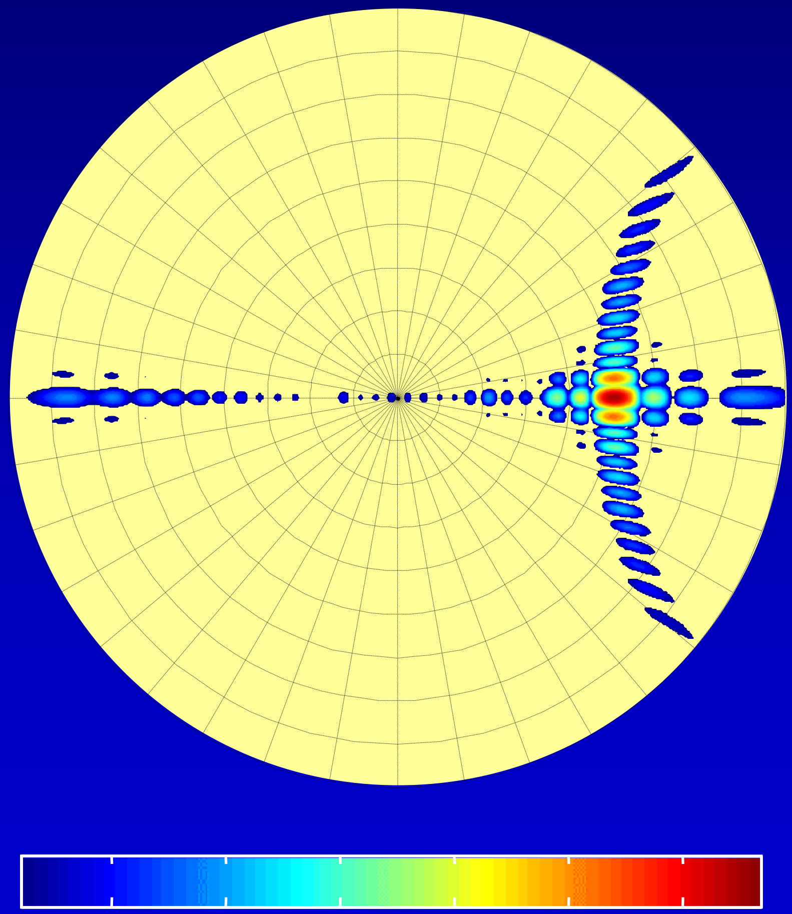
Analysis of large scanning arrays

f = 19.50 GHz ; ϑ_0 = 40° (elev.)



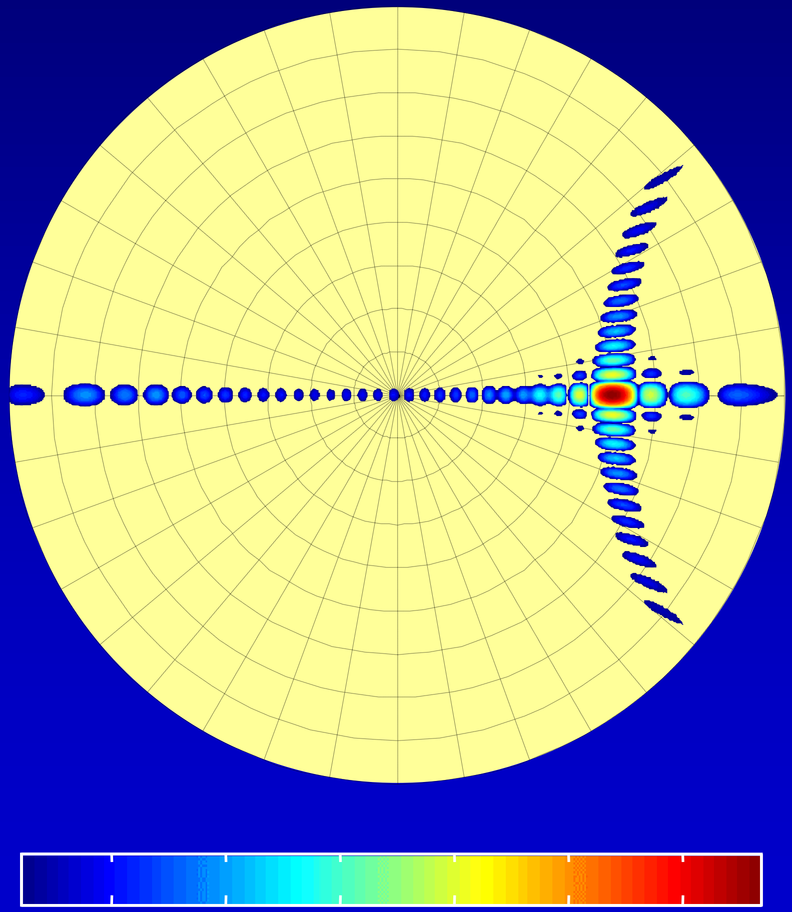
Analysis of large scanning arrays

f = 19.70 GHz ; $\vartheta_0 = 40°$ (elev.)



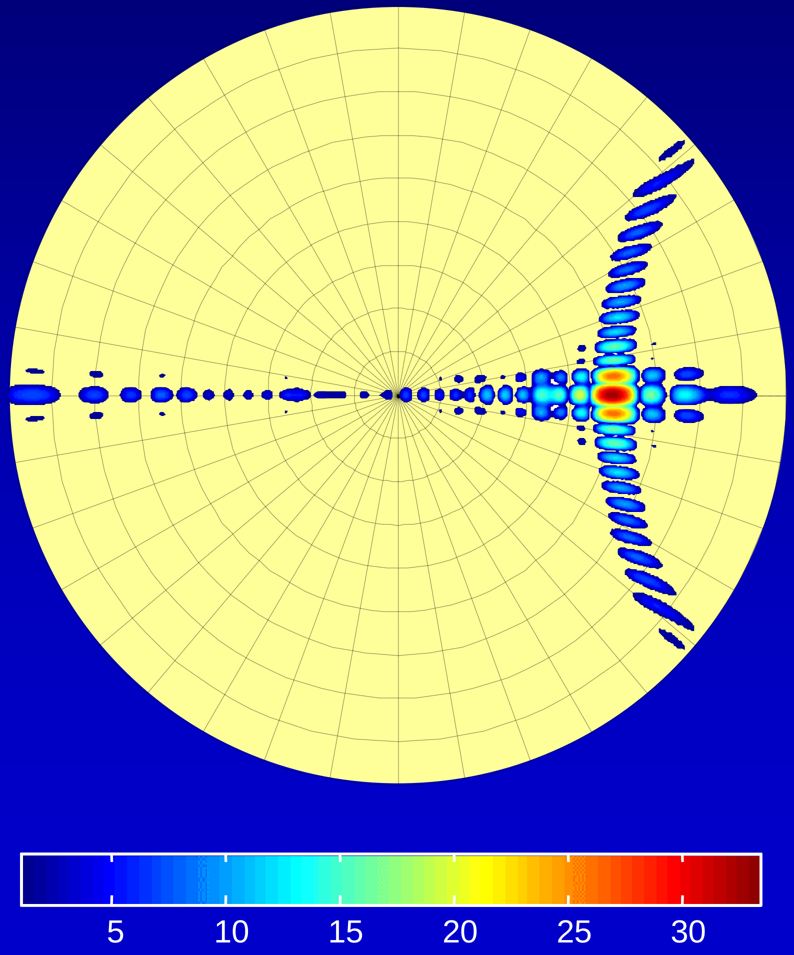
Analysis of large scanning arrays

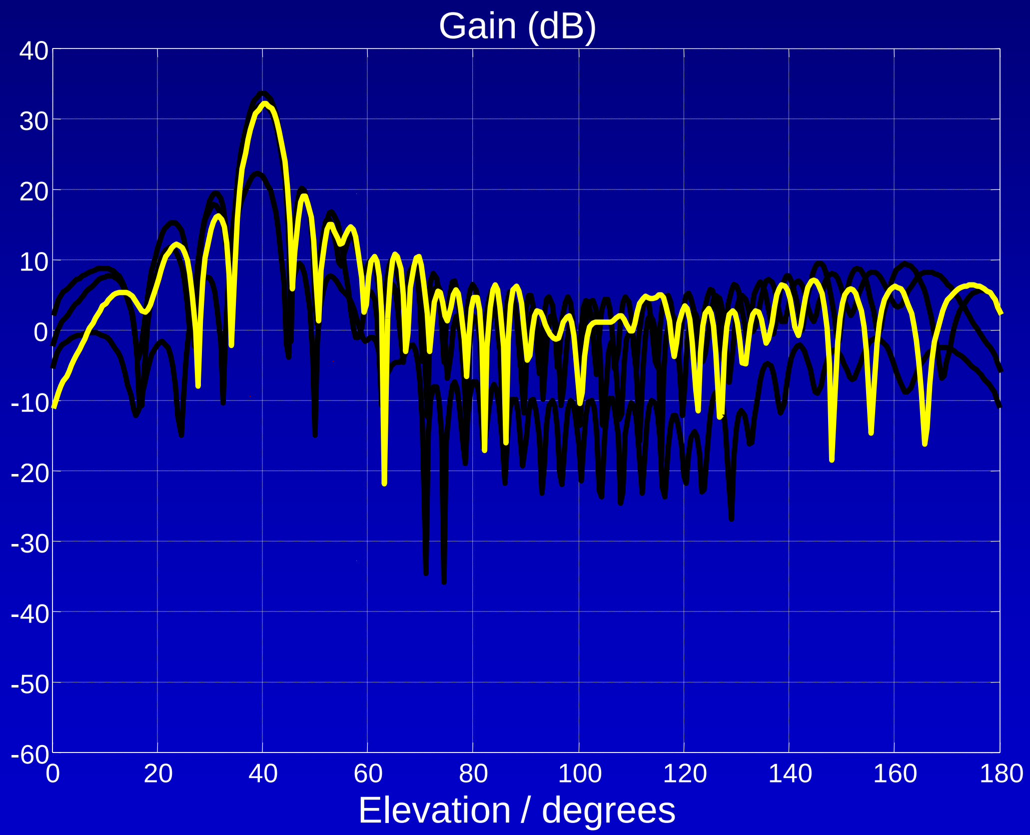
f = 19.95 GHz ; ϑ_0 = 40° (elev.)



Analysis of large scanning arrays

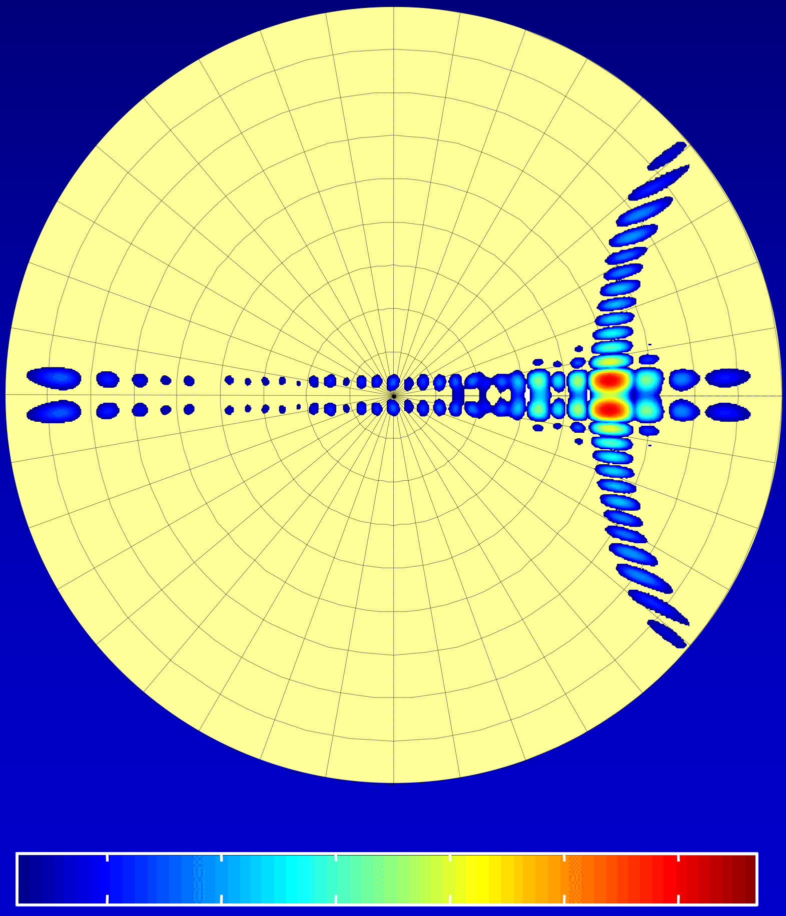
f = 20.20 GHz ; ϑ_0 = 40° (elev.)

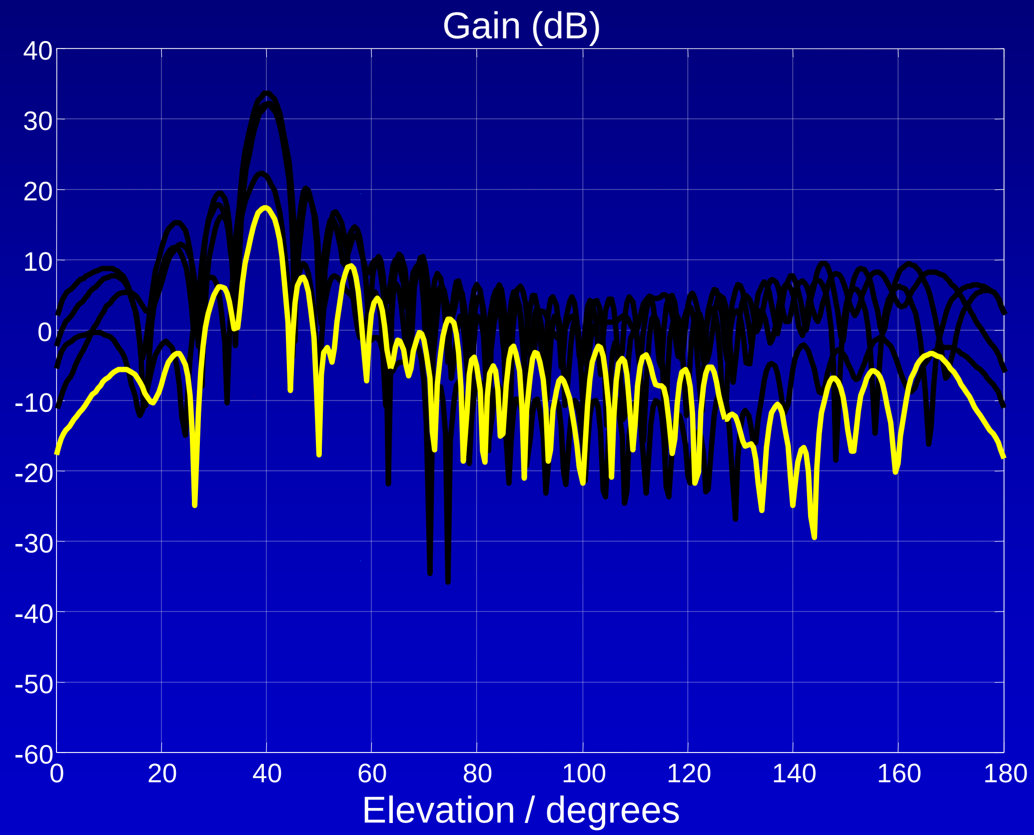




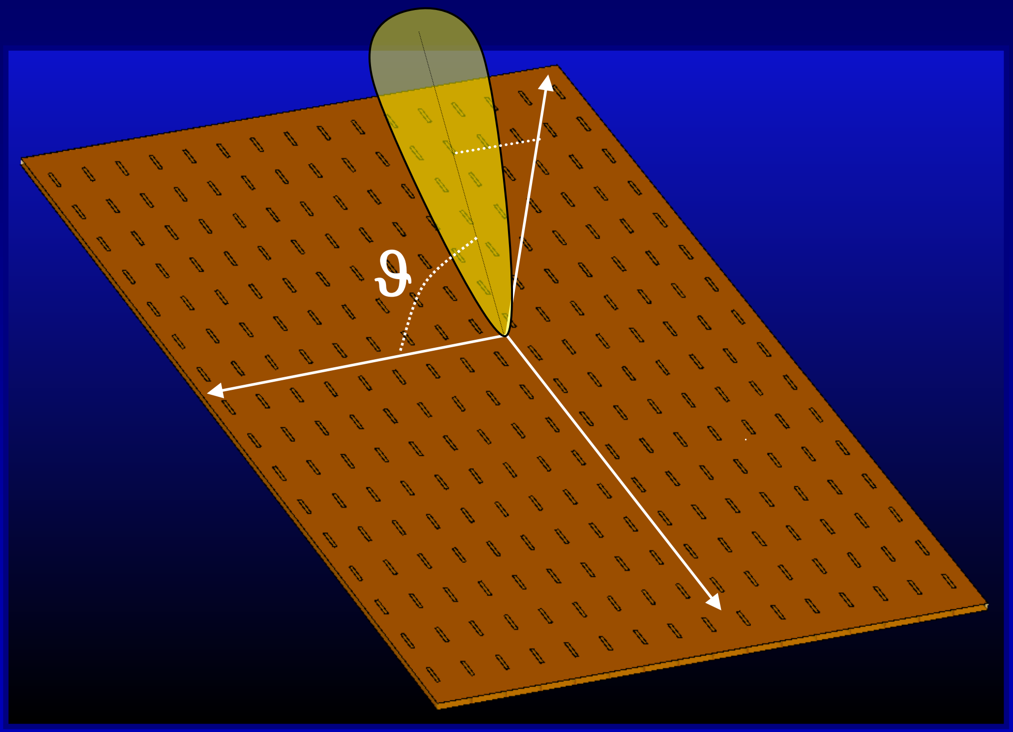
Analysis of large scanning arrays

f = 20.50 GHz ; ϑ_0 = 40° (elev.)





Post-wall slotted WG scanning array



- f_0 = 20 GHz
- ϑ_0 = 40° (elev.)
- 16 subarrays
- 16 slots each sub.
- centre-feeding

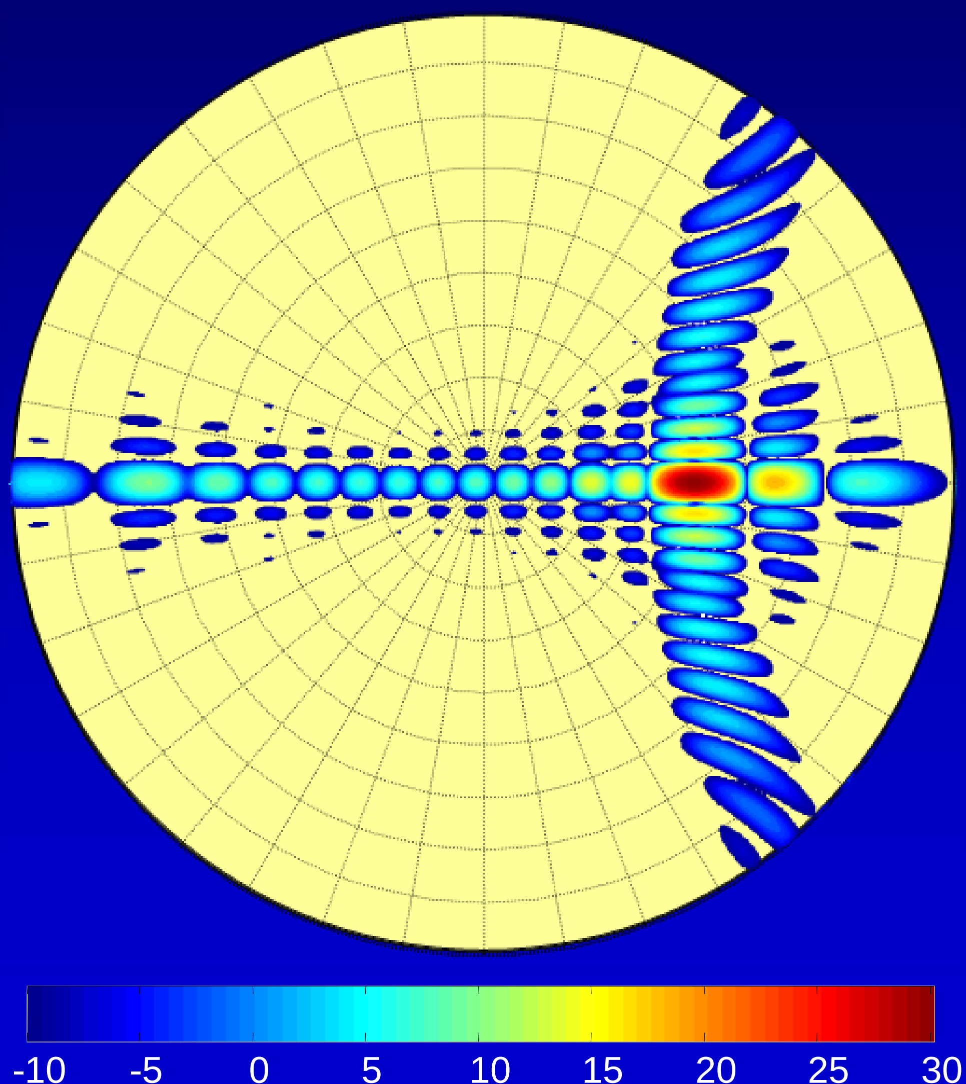
1st Layer: Radiative panel (post-wall slotted WG array)

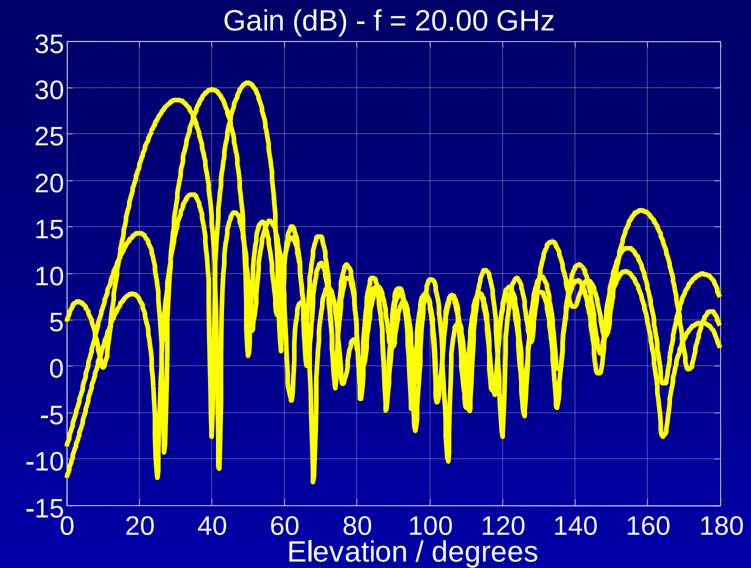
2nd Layer: Feeding network

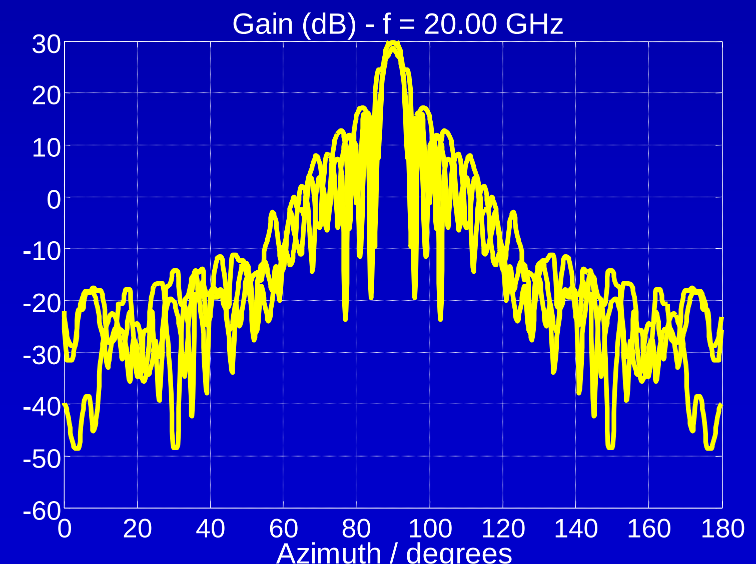
Overall dimensions: 24 cm × 12 cm × 0.2 cm

Results: radiation pattern *vs* angle

$\vartheta_0 = 50° \text{ (elev.)}$

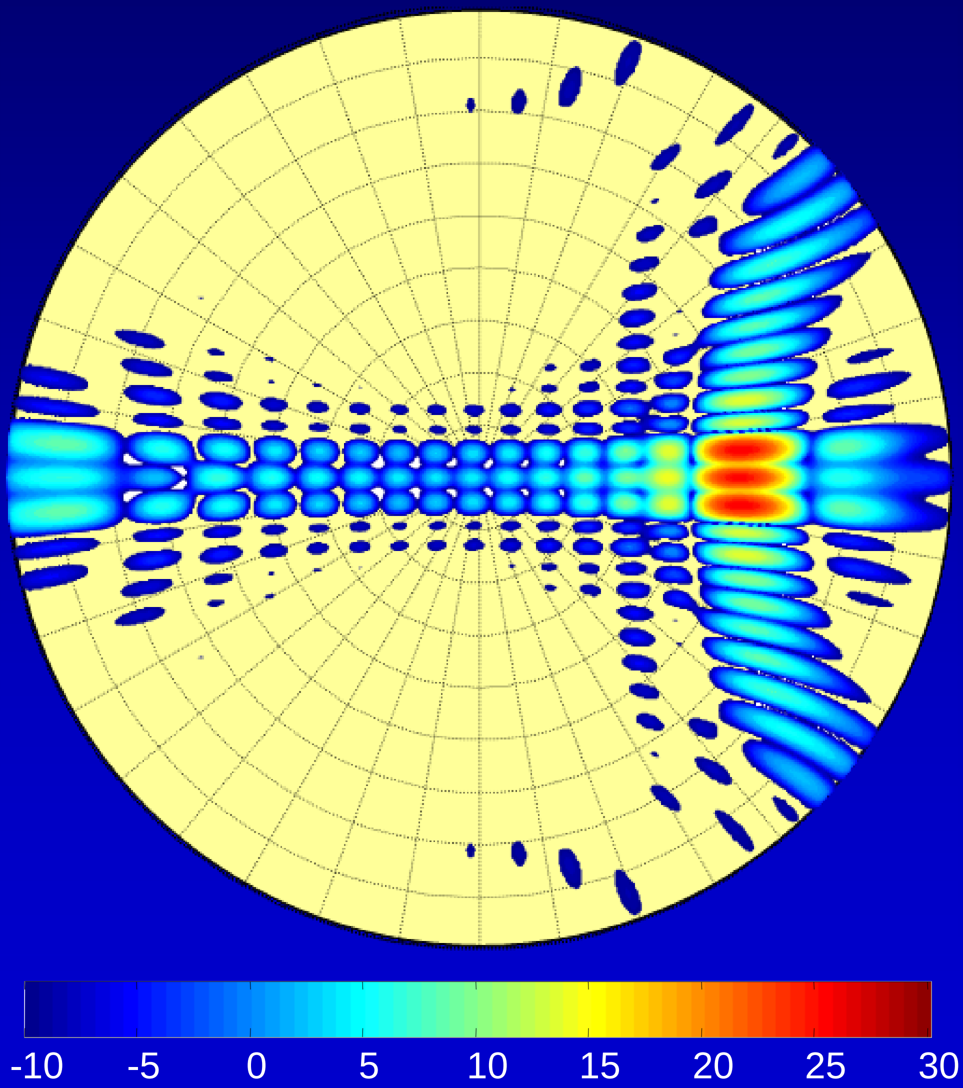


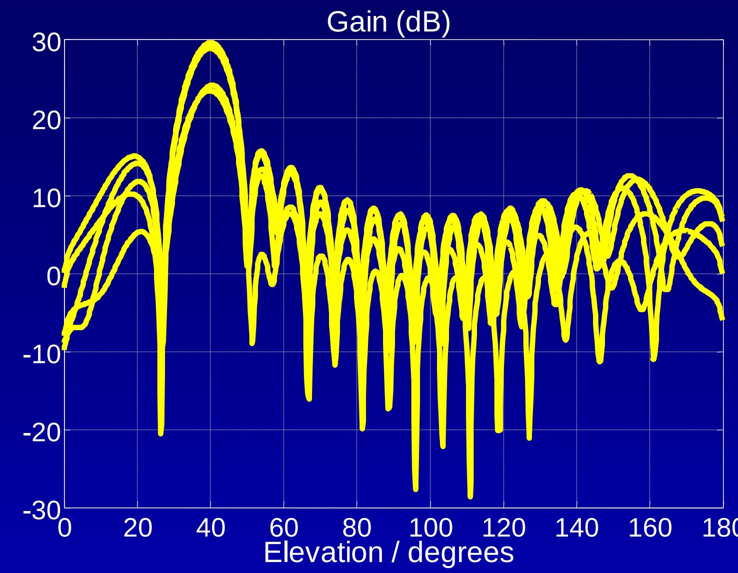


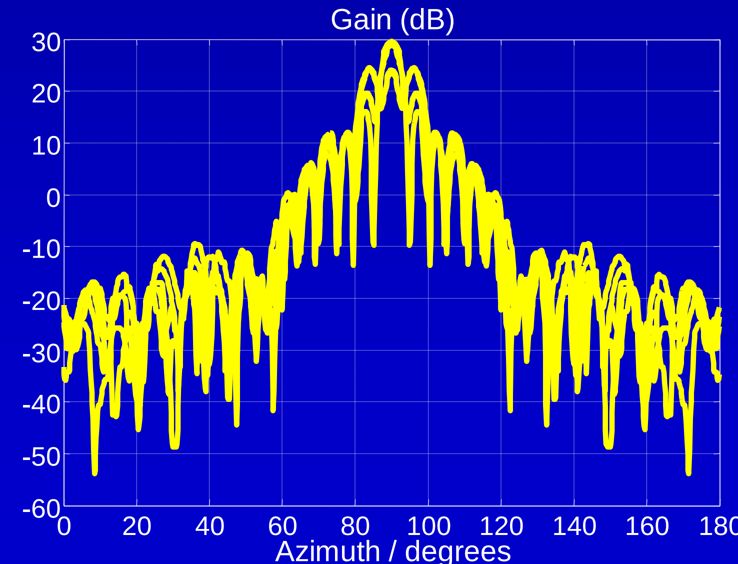


Results: radiation pattern *vs* frequency

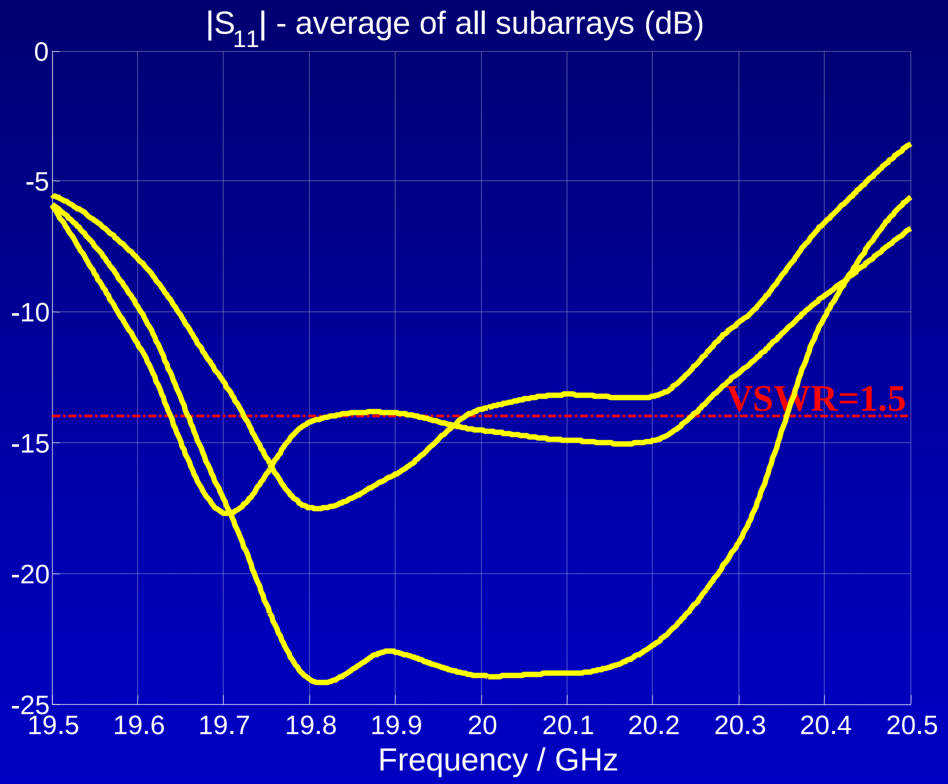
f_0 = 20.00 GHz

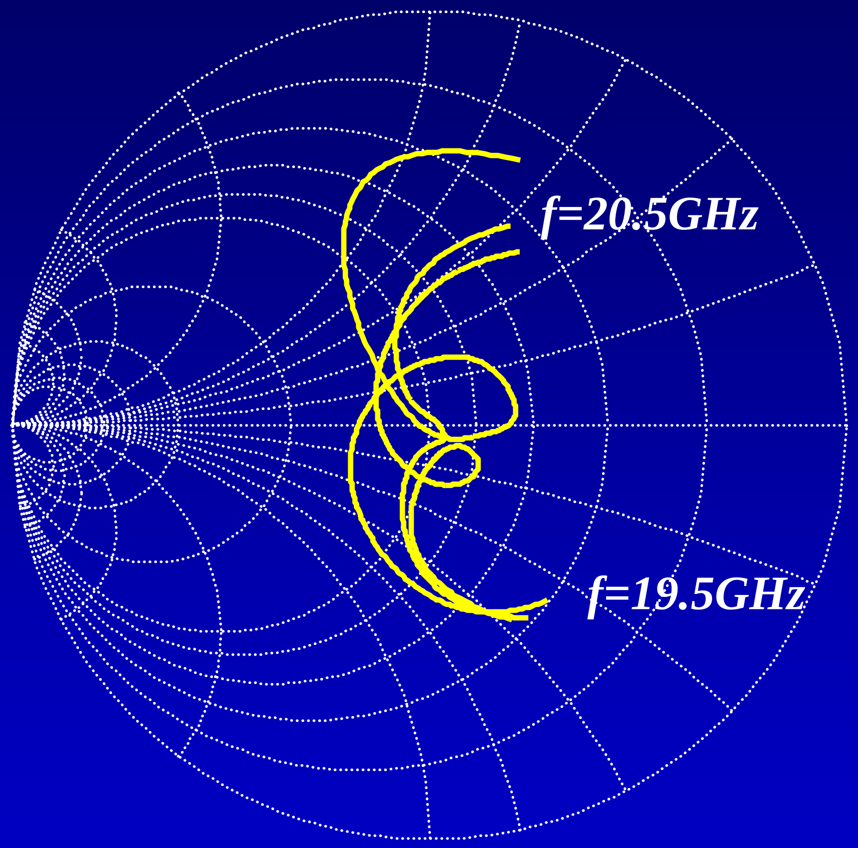






Results: reflection coefficients





$\vartheta_0 = 30°$ (elev.) $\vartheta_0 = 40°$ (elev.) $\vartheta_0 = 50°$ (elev.)

Conclusions

Problem: Analysis and design of large scanning arrays

⇨ Simplified approach based on:

- Equivalent circuit of the radiating element based on accurate full-wave analysis
- Mutual couplings lumped in the model using the “active impedance” concept

⇨ Presented method is applicable to any radiating element with linear current/field distribution

⇩

Fast and accurate analysis and design of large arrays